Reductionism vs Bootstrap:

are things big always made
of things elementary

Slava Rychkov

I.H.E.S. Bures-sur-Yvette
ENS Paris

Occam lecture, Merton College, Oxford 30.04.2018

- Connection between <u>micro</u>scopic & <u>macro</u>scopic
    - particle physics
    - condensed matter
    - statistical physics

How do we think about it ?

↳ Bootstrap

Traditionally:

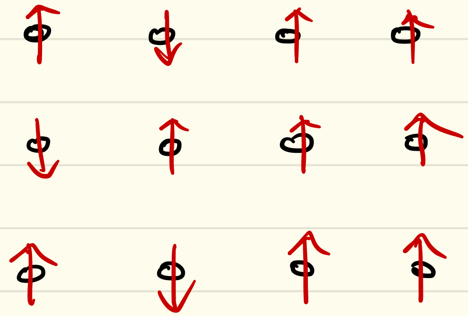
Every phenomenon, or law of physics at distance ℓ,

has origin at some microscopic scale $a \ll \ell$

"Understand" $\approx$ "identify microscopic origin"

MICROSCOPIC MODEL

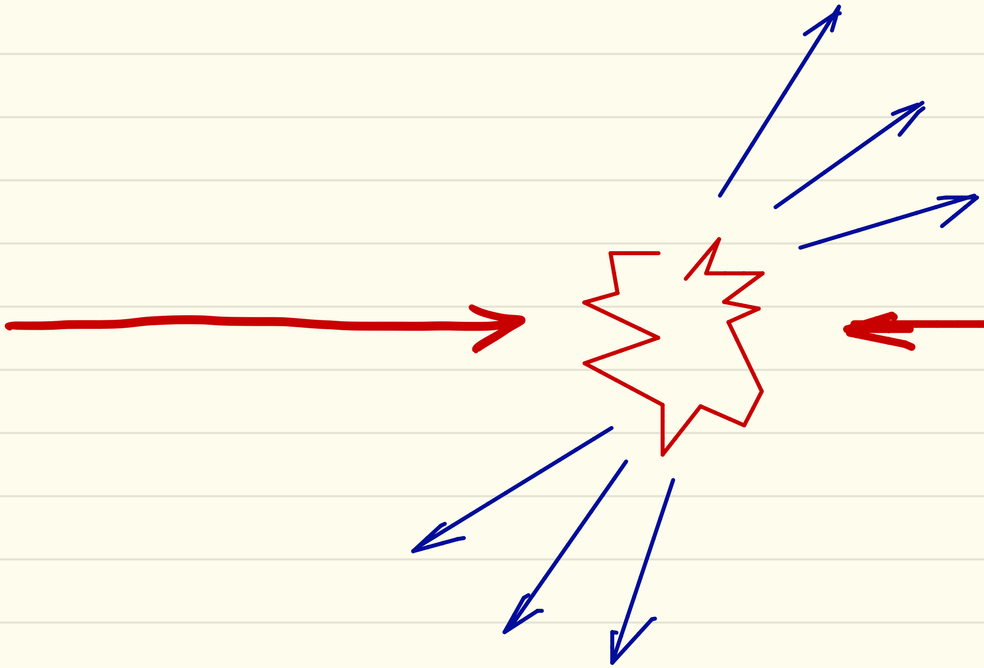
Condensed matter example



individual electron magnetic moments

Particle physics

some other particles are produced



collide some particles
(e.g. protons @ LHC)

What are (microsopic) origins of these processes?

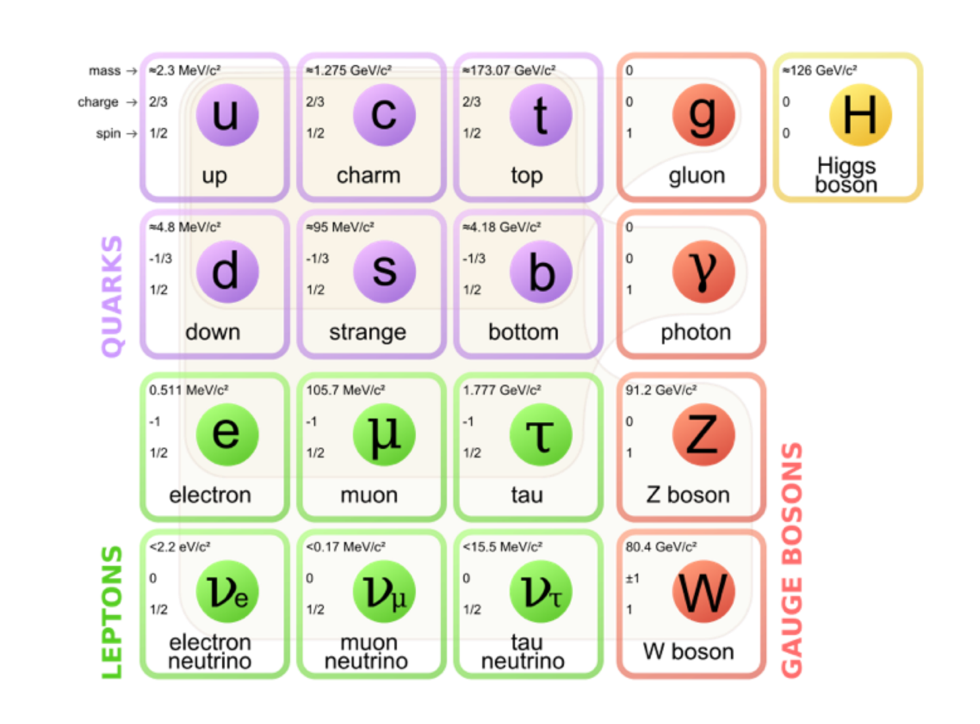
Elementary phenomena:

comfortable to accept them as they are and not look for further explanation

(no rigorous definition)

E.g. electron

Standard Model of Particle Physics



- 17 elementary particles
- Most unstable (cf. unstable nuclei in Mendeleev table)

Composite particles

Particle Data Group

Meson Summary Table

See also the table of suggested $q\bar{q}$ quark-model assignments in the Quark Model section.
• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\rho_3(1690)$	$1^+(3^{--})$
• π^0	$1^-(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• η	$0^+(0^{-+})$	$a_2(1700)$	$1^-(2^{++})$
• $f_0(500)$	$0^+(0^{++})$	• $f_0(1710)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\omega(782)$	$0^-(1^{--})$	• $\pi(1800)$	$1^-(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	$f_2(1810)$	$0^+(2^{++})$
• $f_0(980)$	$0^+(0^{++})$	$X(1835)$	$?^?(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$X(1840)$	$?^?(?^{??})$
• $\phi(1020)$	$0^-(1^{--})$	$a_1(1420)$	$1^-(1^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $f_0(1370)$	$0^+(0^{++})$	• $f_2(2010)$	$0^+(2^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$f_0(2020)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $a_4(2040)$	$1^-(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2100)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_2(2150)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	• $\phi(2170)$	$0^-(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_0(2200)$	$0^+(0^{++})$
• $f_0(1500)$	$0^+(0^{++})$	$f_J(2220)$	$0^+(2^{++}$
$f_1(1510)$	$0^+(1^{++})$		or $4^{++})$
• $f'_2(1525)$	$0^+(2^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
$\rho(1570)$	$1^+(1^{--})$	• $f_2(2300)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$f_4(2300)$	$0^+(4^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_0(2330)$	$0^+(0^{++})$
$a_1(1640)$	$1^-(1^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1640)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
• $\eta_2(1645)$	$0^+(2^{-+})$	$a_6(2450)$	$1^-(6^{++})$
• $\omega(1650)$	$0^-(1^{--})$	$f_6(2510)$	$0^+(6^{++})$
• $\omega_3(1670)$	$0^-(3^{--})$		
• $\pi_2(1670)$	$1^-(2^{-+})$	**OTHER LIGHT**	
• $\phi(1680)$	$0^-(1^{--})$	Further States	

STRANGE ($S = \pm1, C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K^*_3(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K^*_0(1950)$	$1/2(0^+)$
$K^*_2(1980)$	$1/2(2^+)$
• $K^*_4(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K^*_5(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D^*_0(2400)^0$	$1/2(0^+)$
$D^*_0(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D^*_2(2460)^0$	$1/2(2^+)$
• $D^*_2(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D^*_J(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D^\pm_s$	$0(0^-)$
• $D^{*\pm}_s$	$0(?^?)$
• $D^*_{s0}(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(2^+)$
• $D^*_{s1}(2700)^\pm$	$0(1^-)$
$D^*_{s1}(2860)^\pm$	$0(1^-)$
$D^*_{s3}(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I(J^P)$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s$/b-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B^*_J(5732)$	$?(?^?)$
• $B^*_2(5747)^+$	$1/2(2^+)$
• $B^*_2(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1, S = \mp1$)

	$I(J^P)$
• B^0_s	$0(0^-)$
• B^*_s	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B^*_{s2}(5840)^0$	$0(2^+)$
$B^*_{sJ}(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

	$I(J^P)$
• B^+_c	$0(0^-)$
$B_c(2S)^\pm$	$0(0^-)$

$c\bar{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi(3823)$	$?^?(2^{--})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)$	$1(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4055)^\pm$	$?(?^?)$
• $X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4200)^\pm$	$?(1^+)$
$X(4230)$	$?^?(1^{--})$
$X(4240)^\pm$	$?^?(0^-)$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

$b\bar{b}$

	$I^G(J^{PC})$
• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $X(10610)^\pm$	$1^+(1^+)$
• $X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

Baryon Summary Table

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the table are not established baryons. The names with masses are of baryons that decay strongly. The spin-parity J^P (when known) is given with each particle. For the strongly decaying particles, the J^P values are considered to be part of the names.

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1730)$	$3/2^+$	*
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	*
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ^+_c	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ^+_c	$1/2^+$	***
Ξ^0_c	$1/2^+$	***
Ξ'^+_c	$1/2^+$	***
Ξ'^0_c	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2970)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω^0_c	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ^+_{cc}		*
Λ^0_b	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
Σ_b	$1/2^+$	***
Σ^*_b	$3/2^+$	***
Ξ^0_b, Ξ^-_b	$1/2^+$	***
$\Xi'_b(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi^*_b(5955)^-$	$3/2^+$	***
Ω^-_b	$1/2^+$	***
$P_c(4380)^+$		*
$P_c(4450)^+$		*

**** Existence is certain, and properties are at least fairly well explored.

*** Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, etc. are not well determined.

** Evidence of existence is only fair.

* Evidence of existence is poor.

- Why can reduce to a collection of elementary building blocks ?

Better question :

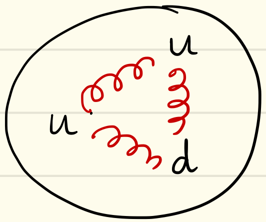
- Could it be otherwise ?

Properties of elementary building blocks in Standard Model:

1. pointlike

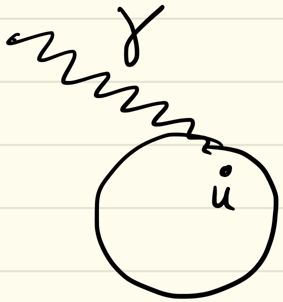
2. weakly interacting
   [at least in some regime]

Proton



- At low energies $p \sim uud + \text{gluons} + \bar{q}q$ pairs
  strongly interacting soup

- At high energies
  quarks and gluons interact
  weakly (asymptotic freedom)



$$Q = \frac{2}{3}e$$

High energy regime – provides 'boundary condition'
– allows to say that quarks exist

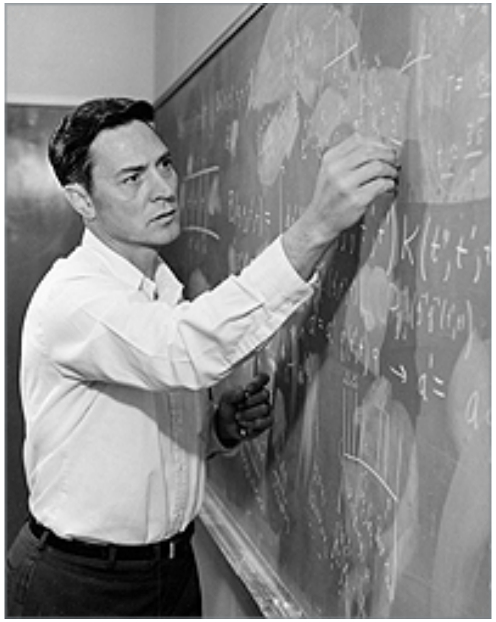
Could it happen that

- proton not pointlike
- and yet no elementary weakly interacting constituents

?

“Nuclear democracy”

Geoffrey Chew, 1960



What if all, infinitely many, particles are equally fundamental?

(or equally composite)

where do you start?

Theory based on consistency

- **Traditional theory:**

  if you see a particle A and particle B
  it's because B is made of A
  or because both B and A are made of something else.

- **In Chew's theory:**

  if you see a particle A and particle B
  it's because **A could not exist without B and B could not exist without A.**

Axioms. "BOOTSTRAP"

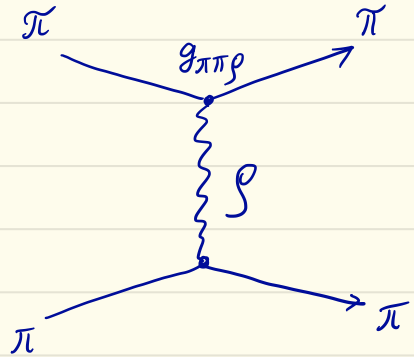
Mathematics analogy: prime numbers

2, 3, 5, 7, 11, 13,…

- infinitely many
- equally fundamental
- elementary constituents:
  every number is a product of primes

ρ - meson via bootstrap

- ρ - meson mediates an attractive force between pions:



- so a system of two pions may form a bound state
- identify bound state with ρ

$\Rightarrow$ gives an equation for $g_{\pi\pi\rho}$, m_ρ

Demise of "Nuclear democracy"

- After 10 years, it was understood that Chew's idea for particle physics was not correct

- Elementary constituents do exist. Quarks were experimentally observed in ~1970

Microscopic approach won.

Philosophically…

PART II

Bootstrap in theory of

CRITICAL POINTS

(= CONTINUOUS PHASE TRANSITIONS)

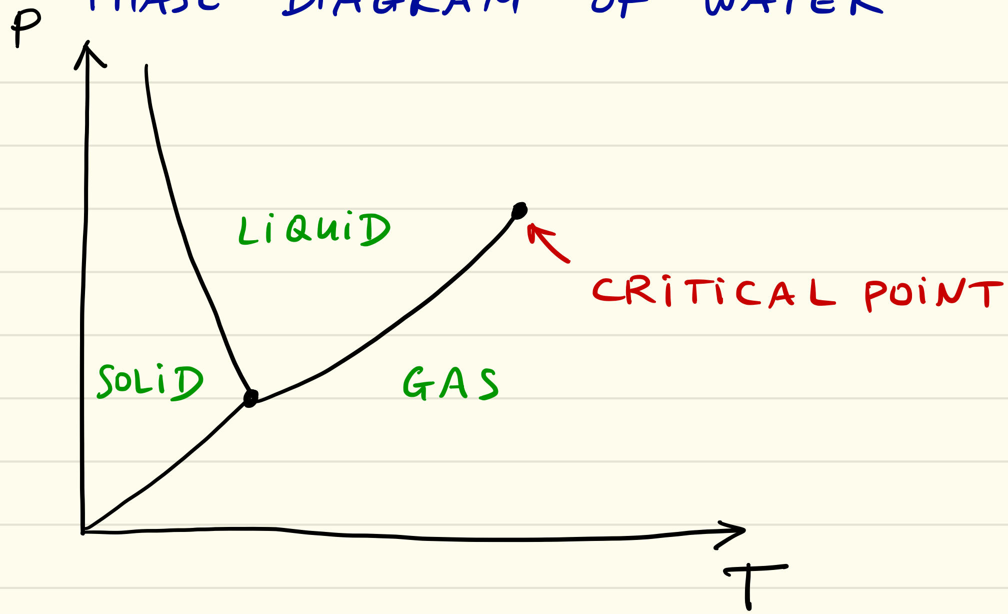
Discontinuous transitions are more common

ice melting

water boiling

These are **discontinuous** transitions
(1st order)

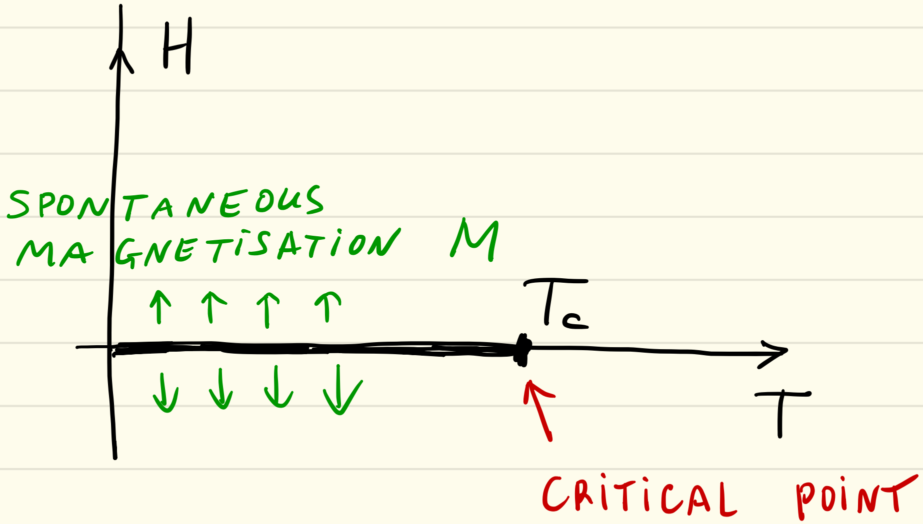
PHASE DIAGRAM OF WATER



$$\rho_{liquid} - \rho_{gas} \to 0$$

<u>CONTINUOUS</u>
PHASE TRANSITION

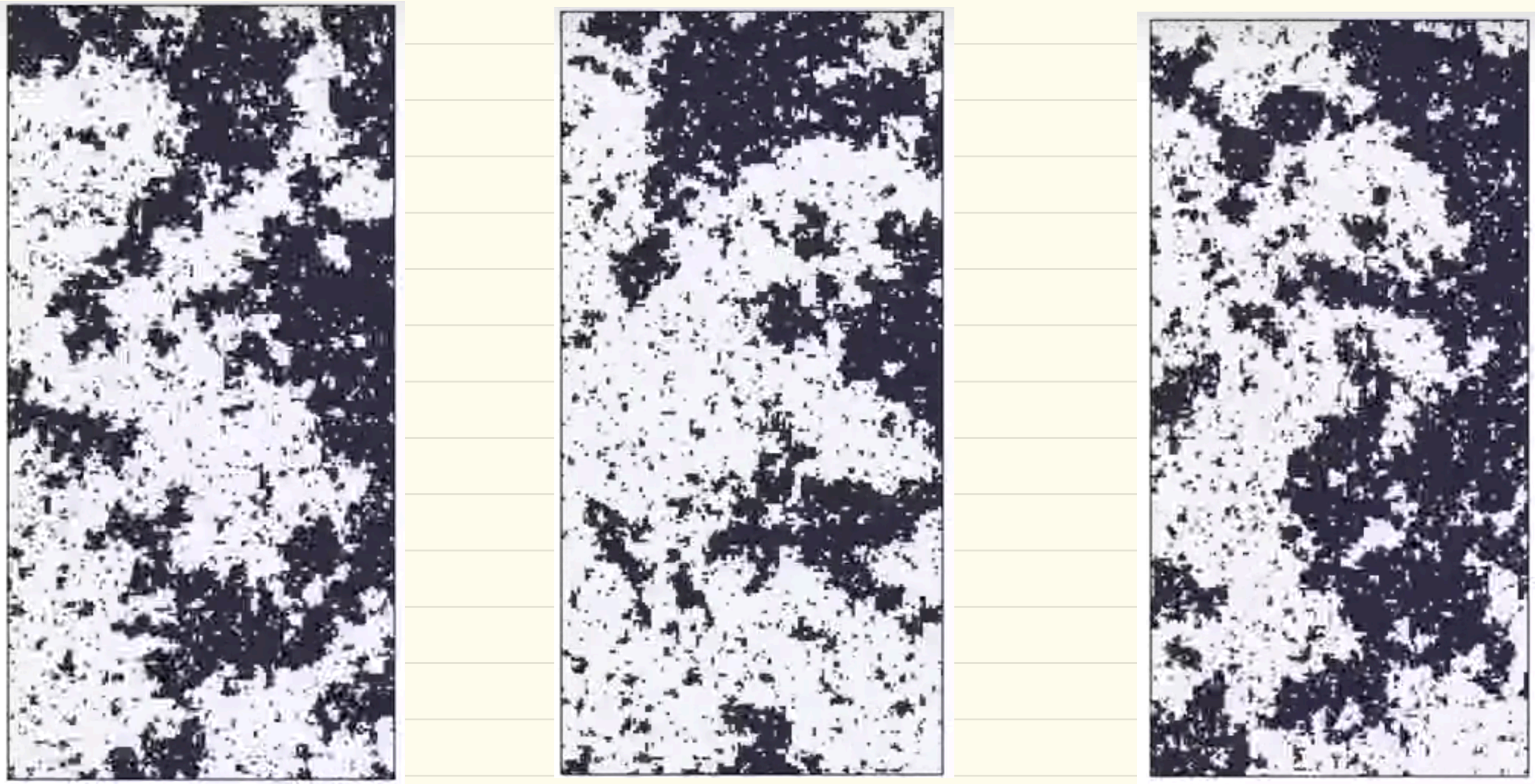
PHASE DIAGRAM OF UNIAXIAL 3D MAGNET
(3D ISING MODEL)



$$M \to 0$$

FLUCTUATIONS AT ALL SCALES

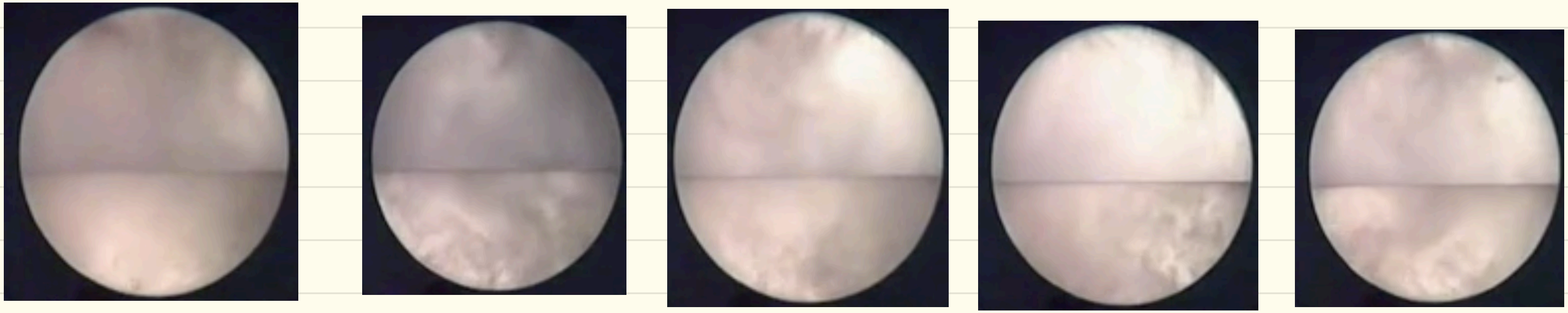
• MAGNETISATION FLUCTUATIONS :
(SNAPSHOTS OF A SIMULATION AT 3 DIFFERENT TIMES)



- ABSENCE OF A CHARACTERISTIC SCALE
- SCALE (& CONFORMAL) INVARIANCE

• DENSITY FLUCTUATIONS IN LIQUID

NEAR CRITICAL POINT:



AT CRITICAL POINT:

CRITICAL OPALESCENCE

LIGHT SCATTERS ON FLUCTUATIONS

CORRELATION FUNCTIONS FOR DENSITY FLUCTUATIONS

average over time (or over thermal ensemble)

$$\langle \delta\rho(r) \rangle = 0$$

deviation from average density

$$\langle \delta\rho(\vec{r}_1)\, \delta\rho(\vec{r}_2) \rangle \sim \frac{\text{const}}{|\vec{r}_1 - \vec{r}_2|^{2\Delta}} \qquad (|\vec{r}_1 - \vec{r}_2| \gg a)$$

intermolecular distance

critical exponent

$$\Delta = 0.5181489(10)$$

UNIVERSAL (SAME FOR ALL LIQUIDS)

SIMILARLY IN MAGNETS:

$$\langle M(\vec{r}_1)\, M(\vec{r}_2) \rangle \sim \frac{\text{const}}{|\vec{r}_1 - \vec{r}_2|^{2\Delta}}$$

SAME Δ!

- CRITICAL POINT OF WATER
  = CRITICAL POINT OF UNIAXIAL MAGNET

- CONFORMAL FIELD THEORY —
  RIGID MATHEMATICAL STRUCTURE BEHIND THIS
  'CRITICAL UNIVERSALITY'

MORE PRECISELY :

$$\langle \delta\rho(\vec{r}_1)\,\delta\rho(\vec{r}_2)\rangle = \sum_{i=1}^{\infty} \frac{C_i}{|\vec{r}_1 - \vec{r}_2|^{2\Delta_i}}$$

Δ_1:

$\mathbb{Z}_2$	ℓ	Δ
−	0	0.5181489**(10)**
−	0	5.2906(11)
−	2	4.180305(18)
−	2	6.9873(53)
−	3	4.63804(88)
−	4	6.112674(19)
−	5	6.709778(27)

. . .

$\mathbb{Z}_2$	ℓ	Δ
+	0	1.412625**(10)**
+	0	3.82968(23)
+	0	6.8956(43)
+	0	7.2535(51)
+	2	3
+	2	5.50915(44)
+	2	7.0758(58)
+	4	5.022665(28)
+	4	6.42065(64)
+	4	7.38568(28)
+	6	7.028488(16)

. . .

Infinitely many 'eigenfluctuations' into which $\delta\rho$ can be decomposed

Two strategies

- TRADITIONAL

Identify a few 'elementary' fluctuating quantities to which everything else is reduced

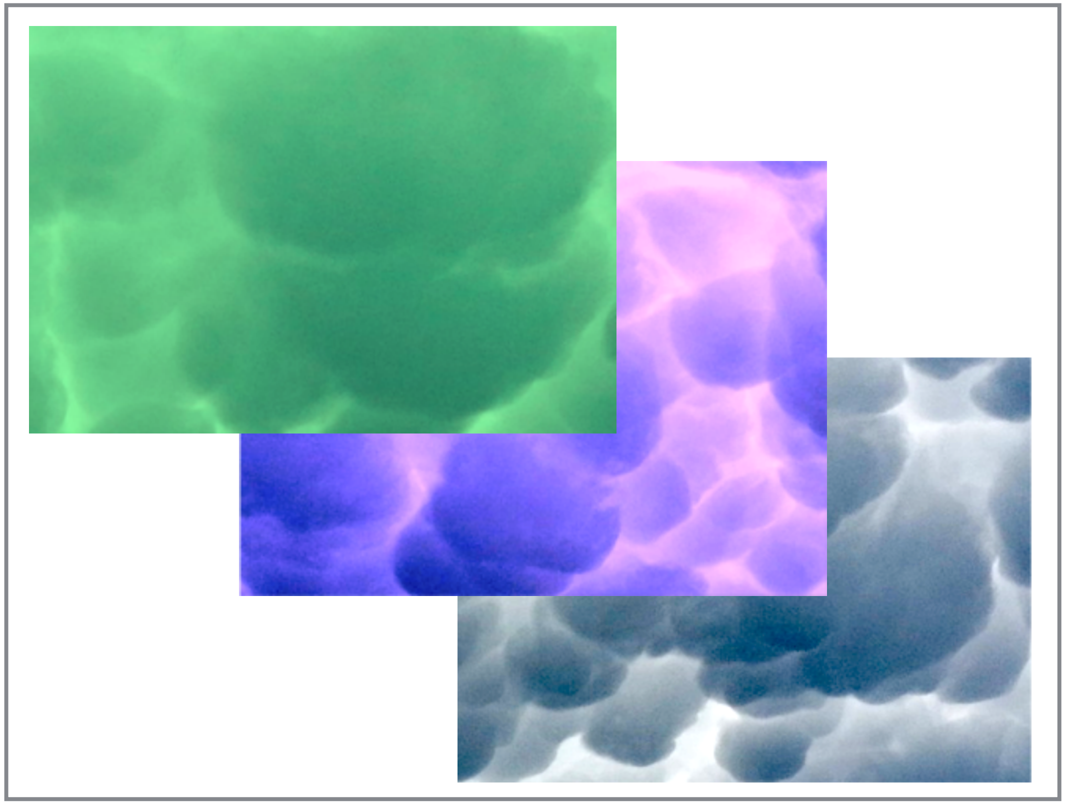
- BOOTSTRAP

All fluctuation types are equally fundamental

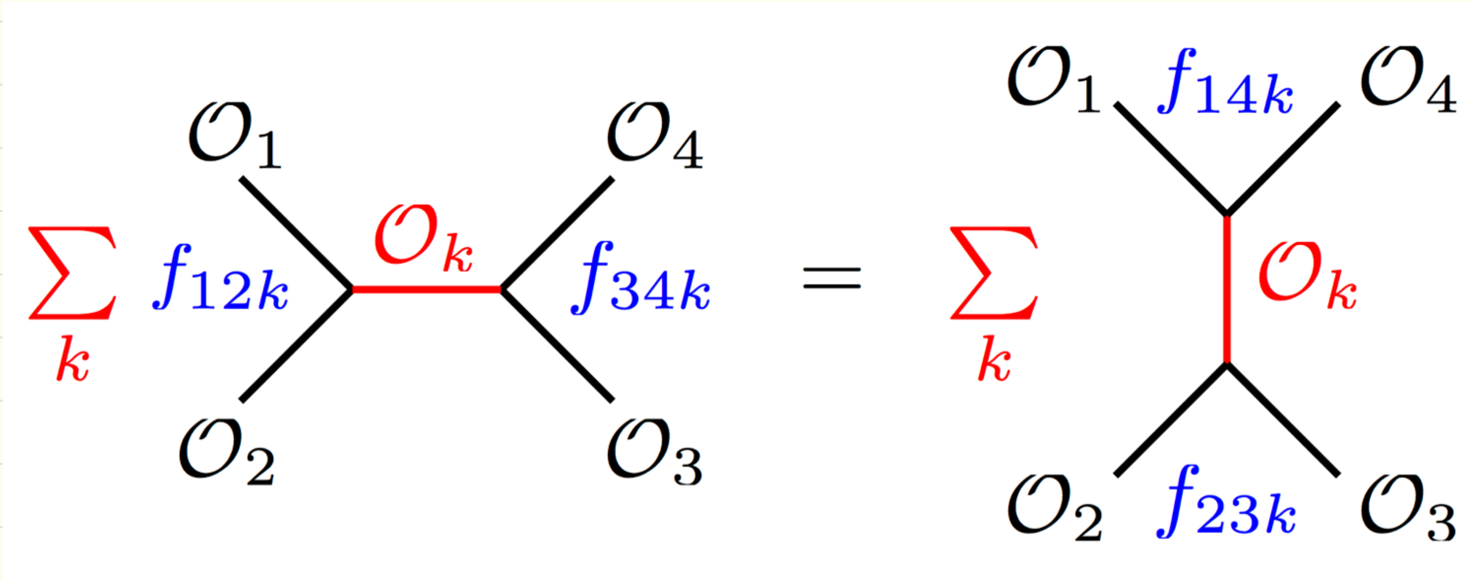
↳ CONSISTENCY EQUATIONS FOLLOW FROM CONFORMAL FIELD THEORY

Infinitely many fluctuations types

Continuous transition =
infinitely many superimposed fluctuating “layers”



Each layer = separate fluctuation type, **all equally fundamental**



$\sum_k$
f_{12k}
$\mathcal{O}_1$
$\mathcal{O}_2$
$\mathcal{O}_k$
$\mathcal{O}_4$
$\mathcal{O}_3$
f_{34k}
$=$
$\sum_k$
$\mathcal{O}_1$
f_{14k}
$\mathcal{O}_4$
$\mathcal{O}_k$
$\mathcal{O}_2$
f_{23k}
$\mathcal{O}_3$

Bootstrap at work

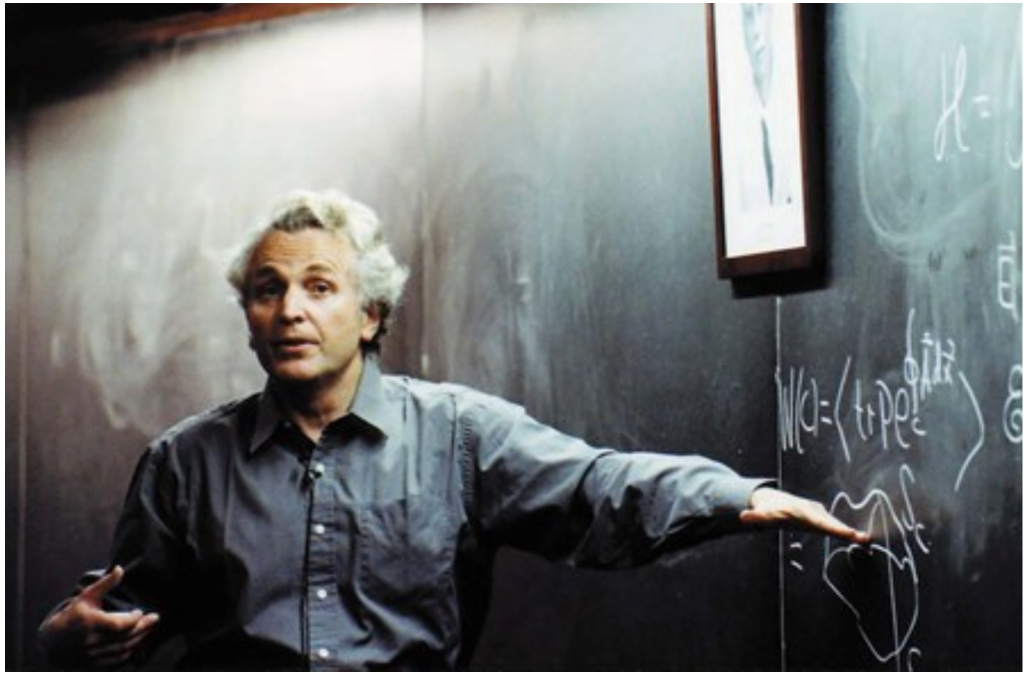
Infinitely many fluctuations

+ consistency conditions $\longrightarrow$ Experimentally verifiable predictions

Chew's ideas found second life in condensed matter physics

Alexander Polyakov

(Gauge Fields and Strings, 1987)



“The garbage of the past often becomes
the treasure of the present
(and vice versa)”